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THE DISTURBANC CAUSED IN HEAT FLOW DUE TO THE PRESENCE OF A GRIFFITH-CRACK IN AN ISOTROPIC RECTANGLE

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ABSTRACT:The closed form expression of temperature distribution in a rectangular isotropic under steady heat conduction is being obtained by using the principle of cross-linear-superposition along with Fourier series. It is found that temperature distribution at crack tips is smooth. Flux possesses Cauchy type singularity at crack tips.

KEYWORDS: [1] Flux intensity factor (FIF) [2] Fourier Series [3] Cross-linear-Superposition

1. INTRODUCTION

The finite dimensional rectangular plates are very commonly used in aero-space vehicles, nuclear reactors or in container where chemical reactions take place. These plates face thermal /heat variations. Discontinuities may be developed in the plates after the use of structure.

We consider a three dimensional body in the form of parallelopiped. The heat distributions in cross-section normal to z axis is similar i.e. there is no change in heat due to z-variable. Therefore we consider a cross-section having one Griffith crack along x-axis and y-axis is passing through the middle of center line of crack.

Chandra [1] had solved few problems of heat distribution is strip. Saroj [2] had extend the heat problems to rectangle rigidly lubricated and with heated wedge also. Sorout [3] had extend the problem to orthotropic medium with heated wedge. Singh [4] solved the thermo-elasticity problem due to heated wedge in an isotropic rectangle with edges parallel to crack line as stress-free.Harendra [5] had solved the thermal stress problem due to heat sources or sinks in orthotropic infinite medium. The problem of [5] was extended to stress-free (parallel to crack line) edges infinite strip by Anjana [6].Sunil [7] had solved the problem of heat distribution in an enclosed cylinder having discontinuities at common interface. Parihar[8] had solved some Triple trigonometrical series and their applications.The title problem is translated to mathematical model through the mixed boundary conditions. We consider a rectangle of length 2a and width $(\delta_1+\delta_2)$. We take x-axis along a line which takes $\delta_1=\delta_2=0$ and y-axis through the middle of length 2a.

$$\frac{\partial T}{\partial y}(x,\delta 1) = Q_1(x), \quad \frac{\partial T}{\partial y}(x,-\delta 2) = Q_2(x), \quad -a\le x\le a \tag{1.1}$$

$$\frac{\partial T}{\partial x}(a,y>0)=Q_3(y), \ \frac{\partial T}{\partial x}(-a,y>0)= Q_4(y), \quad 0\le y\le \delta_1 \tag{1.2}$$

$$\frac{\partial T}{\partial x}(a,y<0)=Q_5(y) , \frac{\partial T}{\partial x}(-a,y<0)=Q_6(y), \quad 0\le y\le \delta_2 \tag{1.3}$$

$$\frac{\partial T}{\partial y}(x,0^+) = Q_7(x)\ ,\ \frac{\partial T}{\partial y}(x,0^-) = Q_8(x), \qquad -b < x < b \qquad (1.4)$$

And continuity conductions across y =0

$$\frac{\partial T}{\partial y}(x,0^+) = \frac{\partial T}{\partial y}(x,0^-), \qquad b \le |x| \le a \qquad (1.5)$$

$$T(x,0^+) = T(x,0^-), \qquad b \le | x | \le a \qquad (1.6)$$

Where $\{0^{\pm}\}$ means the values of functions towards y>0 and y<0, respectively

The heat distribution is steady and it is such that constants of specific heat and linear expansion do not vary with heat change. The heat conduction in solid in two dimension is governed through Laplace's equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T(x,y) = 0, \qquad (1.7)$$

The solution of (1.7) with boundary conditions (1.1)-(1.3) is simple distribution of heat in rectangle. The inclusion of boundary conditions (1.4)-(1.6) in the solution of (1.7) is disturbance caused in heat distribution due to Griffith-crack. $y = 0, 0 \le | x | \le b.$

The plan of the paper is as follows. In section 2 we reduce the problem to dual series equation. The solutions of dual series, which gives Fredholm integral equations, in section 3. Section 3 solves the Fredholm integral equation also. Temperature and Flux are evaluated in section 4.

2. REDUCTION TO AND SOLUTION OF DUAL SERIES EQUATION (SOLUTION)

To solve (1.7) we follow the principle of cross-linear-superposition. We use Fourier series method also. We assume solutions of (1.7) as, for $y > 0.$

$$T(x,y) = \frac{A_0 + C_0}{2} + \sum_{n=1}^{\infty}\left[A_n \cosh\alpha_n y + B_n \sinh\alpha_n y\right]\cos\alpha_n(x) + \sum_{m=1}^{\infty}\left[C_m \cosh\beta_m x + D_m \sinh\beta_m x\right]\cos\beta_m y \qquad (2.1)$$

And for $y < 0$

$$T(x,y) = \frac{E_0 + G_0}{2} + \sum_{n=1}^{\infty}\left[E_n \cosh\alpha_n y + F_n \sinh\alpha_n y\right]\cos\alpha_n x + \sum_{\ell=1}^{\infty}\left[G_\ell \cosh\gamma_\ell x + H_\ell \sinh\gamma_\ell x\right]\cos\gamma_\ell y \qquad (2.2)$$

With

$$\alpha_n = \frac{n\pi}{a}\ ,\ \beta_m = \frac{m\pi}{\delta_1}\ ,\ \gamma_\ell = \frac{\ell\pi}{\delta_2} \qquad (2.3)$$

Where A_n , B_n , C_m , D_m , E_n , F_n , G_ℓ and H_ℓ are eight constants to be determined through conditions (1.1)—(1.6) .

DETERMINATION OF CONSTANTS

The boundary conditions in (1.1)—(1.3) , after using (2.1) and (2.2) respectively, gives

$$A_n a_1 + B_n a_2 = C_1, -E_n a_3 + F_n a_4 = C_2 \tag{2.4}$$

$$C_m a_5 + D_m a_6 = C_3, -C_m a_7 + D_m a_8 = C_4 \tag{2.5}$$

$$G_\ell a_9 + H_\ell a_{10} = C_5, -G_\ell a_{11} + H_\ell a_{12} = C_6 \tag{2.6}$$

Where

$$\begin{aligned} &a_1 = \alpha_n \sinh \alpha_n \delta_1, \quad a_2 = \alpha_n \cosh \alpha_n \delta_1, \quad a_3 = \alpha_n \sinh \alpha_n \delta_2, \quad a_4 = \alpha_n \cosh \alpha_n \delta_2 \\ &a_5 = \beta_m \sinh \beta_m a, \quad a_6 = \beta_m \cosh \beta_m a, \quad a_7 = a_5, \quad a_8 = a_6 \\ &a_9 = \gamma_\ell \sinh \gamma_\ell a \ , \quad a_{10} = \gamma_\ell \cosh \gamma_\ell a \ , \quad a_9 = a_{11}, \quad a_{12} = a_{10} \end{aligned} \tag{2.7}$$

$$\begin{aligned} &C_1 = \int_{-a}^{a} Q_1(x) \cos \alpha_n x dx \ , \quad C_2 = \int_{-a}^{a} Q_2(x) \cos \alpha_n x dx \\ &C_3 = \int_{0}^{\delta_1} Q_3(y) \cos \beta_m y dy \ , \ C_4 = \int_{0}^{\delta_1} Q_4(y) \cos \beta_m y dy \\ &C_5 = \int_{0}^{\delta_2} Q_5(y) \cos \gamma_\ell y dy \ , \ C_6 = \int_{0}^{\delta_2} Q_6(y) \cos \gamma_\ell y dy \end{aligned} \tag{2.8}$$

Thus six constants out of eight constants will be determined. Further , the conditions (1.4)—(1.5) will gives us,

$$\alpha_n (B_n - F_n) = \int_{0}^{a} \langle Q_7(x) - Q_8(x) \rangle \cos \alpha_n x dx = C_7 \tag{2.9}$$

Now using (1.6) to give

$$\begin{aligned} &T(x,0^+) - T(x,0^-) = 0, \qquad b \le x \le a \\ &\frac{B_0}{2} + \sum_{n=1}^{\infty} \left(\frac{a_4}{a_3} + \frac{a_2}{a_1} \right) \cos \alpha_n x = P_1(x), \qquad b \le x \le a \end{aligned} \tag{2.10}$$

Now we calculate

$$\frac{\partial T}{\partial y}\left(x,0^+\right) + \frac{\partial T}{\partial y}(x,0^-) = Q_7(x) + Q_8(x) = Q_9(x), \qquad 0 \le x \le b$$

$$\sum \alpha_n B_n \cos\alpha_n x = \frac{1}{2}Q_9(x) + \frac{1}{2}\sum \alpha_n \frac{C_7}{\alpha_n}\cos\alpha_n dx$$
$$= Q_{10}(x) \quad , \qquad 0 \le x \le b \qquad (2.11)$$

$$P_1 = \sum_{\ell=1}^{\infty}\left[\cosh(\gamma_\ell x).\frac{C_5 - C_6}{2a_9} + \sinh(\gamma_\ell x).\frac{C_5 + C_6}{2a_{10}}\right] - \sum_{m=1}^{\infty}\left[\frac{C_3 - C_4}{2a_5}\cosh(\beta_m x) + \frac{C_3 + C_4}{2a_6}\sinh(\beta_m x)\right] - \sum_{n=1}^{\infty}\left[\frac{C_1}{a_1} + \frac{C_8}{\alpha_n a_3}\right]\cos\alpha_n x$$

$$C_8 = C_2\alpha_n + C_9 a_4 \qquad (2.12)$$

Now we take

$$B_n\left(\frac{a_1a_4 + a_2a_3}{a_1a_3}\right) = \varphi_n \qquad \varphi_0 = B_0 \qquad (2.13)$$

Then (2.10)-(2.11) change to

$$\frac{\varphi_0}{2} + \sum_{n=1}^{\infty}\varphi_n \cos(\alpha_n x) = P_1(x) \qquad , \; b \le x \le a \qquad (2.14)$$

$$\sum_{n=1}^{\infty}\alpha_n\varphi_n \cos\alpha_n x = \theta_{11}(x), \qquad 0 \le x \le b$$
$$= \theta_{10}(x) - \sum_{n=1}^{\infty}\alpha_n B_n\left(\frac{a_1a_3 - a_1a_4 - a_2a_3}{a_1a_4 + a_2a_3}\right)\cos\alpha_n x \qquad (2.15)$$
$$Q_{10}(x) = \frac{1}{2}Q_9(x) + \frac{1}{2}\sum C_7\cos(\alpha_n x)$$

Thus the physical problem is reduced to the solution of dual series relations (2.13)-(2.14)

SOLUTION OF DUAL SERIES

The solution of Dual series equation (2.13)-(2.14) will be obtained by the method of Parihar[8]. We assume the trail series as-

$$\alpha_n\varphi_n = 2\left[\int_0^b\left\langle g(t) - \frac{1}{a}\int_b^a P_1^{'}(t)\right\rangle \sin(\alpha_n t)dt\right] \qquad (2.16)$$

$$\varphi_0 = 2\left[\left\langle\int_0^b t.g(t) - \frac{1}{a}\int_b^a P_1^{'}(t)\right\rangle dt.P_1(a)\right] \qquad (2.17)$$

Then the substitution of (2.16)-(2.17) into (2.15) satisfies it identically. We take g(0)=0 with no loss of generality .The substitution of (2.16) into (2.14) and inverting the integral.

We get

$$g(t) = \frac{2\sin(gt/2)}{a^2\sqrt{G(t,b)}}\left[\Delta_0(t) + \int_0^b g(y)K(y,t)dy\right] , \quad 0 \le t \le b \qquad (2.18)$$

Where

$$\Delta_0(t) = \int_0^b \frac{\cos(qy/2)\sqrt{G(y,b)}}{G(t,y)} + \int_b^a P_1(\alpha)\sin q(qx)dx + P_2(y) \qquad (2.19)$$

$$K(y,t) = \int_0^b \frac{\cos(q,\alpha/2)\sqrt{G(\alpha,b)}}{G(\alpha,t)} M(\alpha,y)d\alpha \qquad (2.20)$$

$$M(\alpha,y) = \sum_{n=1}^{\infty} b_1(\alpha_n)\sin(\alpha_n\alpha)\cos(\alpha_n y) \qquad (2.21)$$

$$P_2(y) = \int_b^a P_1^{'}(\alpha)M(\alpha,y)d\alpha \qquad (2.21\text{ a})$$

$$G(t,y) = |\cos(qt) - \cos(qy)| \quad , \quad q = \frac{\pi}{2}$$

Thus the problem is reduced to the solution of Fredholm integral equation of second kind given by (2.18).

3. SOLUTION OF FREDHOLM INTEGRAL EQUATION

The solution of Fredholm integral equation (2.18) will be obtained by the method approximation. Before, we solve it we take boundary conditions as known specific functions.

Let $\theta_1(x) = d_1 =$ constant

$\theta_2(x) = 0 =$ no flux across $y = -\delta_2$

$\theta_3(y) = \theta_4(y) = d_2 =$ Constants (3.1)

$\theta_5(y) = \theta_6(y) = 0$ No flux across $x = \pm a$

$\theta_7(y) = \theta_8(y) = \theta_0(y)$ For y < 0

The use of definitions of C_i , i=1,2,3,….7 and (3.1), we get

$$C_1 = d_1 a, C_2 = 0, C_3 = C_4 = \delta_1 d_2, \; C_5 = C_6 = C_7 = C_8 = 0 \qquad (3.2)$$

Then

$$\theta_9(x) = 2(\theta_0(x)) \quad , \quad \theta_{10}(x) = \theta_0(x) \qquad (3.3)$$

Using (3.2) in (2.12) , we get

$$P_1(x) = -2\sum_{m=1}^{\infty} \frac{d_2\delta_1}{2a_6}\sinh\beta_m x - \frac{ad_1}{2}\sum_{r=0}^{\infty}\frac{1}{t_1}\log|\beta_1^2 - \cos qx| \qquad (3.4)$$

$$t_1 = e^{-(2r+1)q\delta_1/2} \quad , \quad \beta_1^2 = \frac{1+t_1^2}{2t_1^2}$$

$$P_2(x) = \sin(qx)\delta_3(q) + \sin(2qx)\delta_3(2q) \qquad (3.5)$$

$$\delta_3(y) = \frac{1}{4}\sum_{m=1}^{\infty}\sec h\beta_m a\left\langle \frac{2\delta_1(\beta_m,y)}{d_1(r,\delta_1,\delta_2}\right\rangle + \frac{1}{4}\sum_{r=0}^{\infty}\frac{1}{2t_1}\frac{\delta_2(\beta,y)}{d_1(r,\delta_1,\delta_2)} \qquad (3.5\text{ a})$$

$$\delta_1(\beta_m,y) = \frac{2}{\beta_m^2 + y^2}\left[\beta_m\langle\cosh\beta_m b + \cos qb\cosh\beta_m b\rangle\right] + y\sin qb\cosh\beta_m b \qquad (3.5\text{ b})$$

$$\delta_2(\beta_1,y) = -\frac{1+\cos qb}{y} + \frac{\beta_1^2}{y}\left\{\log\left|\frac{\beta_1^2+1}{\beta_1^2 - \cos qb}\right|\right\} \qquad (3.5\text{ c})$$

$$M(\alpha,y)=-\frac{1}{4}\sum_{r=0}^{\infty}2\sin(q\alpha)\cos(qy).d_1(r,\delta_1,\delta_2)+\sin(2q\alpha)\cos(2qy)d_2(r,\delta_1,\delta_2) \qquad (3.6)$$

$$d_1(r,\delta_1,\delta_2)=\frac{1}{\cosh 2qr(\delta_1+\delta_2)}+\frac{1}{\cosh 2q(r(\delta_1+\delta_2)+\delta_1)} \qquad (3.6\ a)$$

$$d_2(r,\delta_1,\delta_2)=\frac{1}{\cosh^2 2qr(\delta_1+\delta_2)}+\frac{1}{\cosh^2 2q(r(\delta_1+\delta_2)+\delta_1)} \qquad (3.6\ b)$$

Thus the Fredholm integral equation (2.18) will have their parts as-

$$\Delta_0(t)=\int_0^b\frac{\cos(qy/2)\sqrt{G(t,b).}}{G(y,t)}\theta_{12}(y)dy \qquad (3.7)$$

$$\theta_{12}(y)=\theta(y)+P_2(y)+\int_b^a\frac{P_1^{'}(x)\sin sx}{G(x,y)}$$

$K(y,t)$ Will be given by (2.19) and first of (2.20)

Now we calculate $K(y,t)$ which is given below

$$K(y,t)=-\frac{1}{4}\sum_{r=0}^{\infty}\left[\ 2d_1(r,\delta_1,\delta_2)\sin(q\alpha)\mathrm{I}_1(t)\ +\ \sin(2q\alpha)d_2(r,\delta_1,\delta_2)\left\langle 2\mathrm{I}_2(t)-\mathrm{I}_0(t)\right\rangle\ \right]\cos pqy \qquad (3.8)$$

Where

$$\mathrm{I}_n(t)=\int_0^b\frac{\cos(qy/2)\sqrt{G(y,b)}}{G(y,t)}\cos^n(qy)dy \qquad (3.8)\ a$$

$$t_n=\frac{\sqrt{2}\pi}{q}\sum_{\ell=0}^{n}(-1)^{\ell}\ {}^nC_\ell G^{\ell+1}(0,b)\frac{\Gamma 2\ell}{2^{2\ell+1}\Gamma\ell\Gamma\ell+2} \qquad (3.8)\ b$$

We consider a further special case of flux prescribed over crack faces. We assume that

$$\theta_0(x)=\theta_0=\text{Constant} \qquad (3.9)$$

Then

$$\Delta_0(t)=\Delta_{01}(t)+\Delta_{02}(t)+\Delta_{03}(t) \qquad (3.10)$$

$$\Delta_{01}(t)=\frac{\theta_0 a}{\sqrt{2}}=\text{Constant}$$

$$\Delta_{02}(t)=b_1d_1\mathrm{I}_1(t)+d_2b_3\left\{2\mathrm{I}_2(t)-\mathrm{I}_0(t)\right\} \qquad (3.10)\ a$$

$$\Delta_{03}(t)=b_2\mathrm{I}_0(t)+\sin(qt)d_1\mathrm{I}_1(t)+\sin(2qt)d_2\left\langle 2\mathrm{I}_2(t)-\mathrm{I}_0(t)\right\rangle \qquad (3.10)b$$

$$b_1=\sum_{m=1}^{\infty}\left[q\cosh(\beta_m a)\left\langle 1+\cos(qb)\right\rangle-\beta_m\sin\beta_m b\sinh\beta_m b\right]\frac{\delta_1 d_2}{a_6} \qquad (3.10)c$$

$$b_2=\sum_{r=0}^{\infty}\sqrt{\beta_1^4}-1\left[\cot^{-1}\left\langle\sqrt{\frac{\beta_1^2+1}{\beta_1^2-1}}.\tan(qb/2)\right\rangle\right] \qquad (3.10)\ d$$

$$b_3=\sum_{r=0}^{\infty}\left[b-a+2b_2\beta_1^2+\frac{\sin 2qb}{2}\right]\frac{ad_1}{2t_1} \qquad (3.10)\ e$$

Now we proceed to solve Fredholm integral equation by assuming, g(t) as-

$$g(t) = \sum_{m=0}^{\infty} g_m(t)\left\{\cosh\left\langle mq(\delta_1+\delta_2)\right\rangle\right\}^{-1} \quad , \ 0 \le t \le b \tag{3.11}$$

Now we use (3.11) in (2.18) and use $\Delta_0(t)$, $K(y,t)$ from (3.10) and (3.8) respectively for the case (3.9) i.e. when constant flux is maintained at crack faces , then compare the co-efficient of $\{\cosh mq(\delta_1+\delta_2\}^{-1}$, for different m= 0,1,2,3.. from both sides of the equations. We retained for m = 0,1,2 only.

$$g(t) = \frac{2\sin(qt/2)\psi_0}{a\sqrt{2G(t,b)}}\left[1+b_2\right] \tag{3.12}$$

$$g_1(t) = 0 \tag{3.13}$$

$$g_2(t) = \frac{2}{a^2}\frac{\sin(qt/2)}{\sqrt{G(t,b)}}[\left[b_1 I_1(t) + b_3\left\{2I_2(t) - I_0(t)\right\}\right] + \ \sin(qt)I_1(t) + 2I_1(t)\int_0^b g_0(\alpha)d\alpha\ ] \tag{3.14}$$

Then

$$g(t) = \frac{2\sin(qt/2)\psi_0}{a^2\sqrt{G(t,b)}}\left[\frac{a}{\sqrt{2}}(1+b_2)\left\{\frac{-a}{\sqrt{2}}.b_3 + I_1(t)\left\langle b_1 + \sin(qt) + 2b_2\right\rangle + 2b_3 I_2(t)\right\}\frac{1}{\cosh^2 2q(\delta_1+\delta_2)}\right], 0 \le t \le b$$

(3.15)

4. TEMPERATURE AND FLUX

The temperature over crack faces will be obtained from the series given by (2.13), after taking $P_1(x)$ right hand side to left hand side for $0 \le x \le b$

Thus

$$T(x,0) = \int_x^b g(t)dt \quad , \qquad 0 \le x \le b \tag{4.1}$$

Where g(t) is given by (3.15). Now using (3.15) in (4.1) and evaluating the integrals, we get

$$T(x) = \frac{\sqrt{2}\psi_0}{a}\left[X_1(x).e_1 + \psi_1(x)e_2 + \psi_2(x).e_3 + \psi_3(x).e_4 + \psi_4(x)e_5\right] \tag{4.2}$$

With

$$e_1 = 1 + b_2 - \left\langle b_3 - \frac{(b_1+2b_2)G(0,b)}{2} + \frac{b_3 G^2(0,b)}{4}\right\rangle e_5$$

$$e_2 = \left\langle \frac{b_1+2b_2}{2} + b_3 G(0,b)\right\rangle e_5$$

$$e_3 = b_3 e_5 \qquad , \qquad e_4 = e_5 G(0,b)$$

$$e_5 = \sec h^2\left\langle 2q(\delta_1+\delta_2)\right\rangle , \ X_1^{(X)} = \cosh^{-1}\left\langle\frac{\cos(qx/2)}{\cos(qb/2)}\right\rangle \tag{4.3)-(4.4}$$

$$\psi_1(x) = \frac{\sqrt{2}X_1}{q}\left[\cos(qb) + \cos^2(qb/2)\right] - \frac{\cos(qx/2)\sqrt{G(x,b)}}{q} \tag{4.5}$$

$$\psi_2(x)=\frac{1}{\sqrt{2}q}\cos^2(qb/2)\left[\begin{array}{l}\cos^2(qb/2)\left\langle X_1(x)-\left\{G(x,b)+\cos^2(qb/2)\right\}\cos(qx/2)\sqrt{G(x,b)}\right\rangle- \\ 2X_1(x)+\sqrt{2}\cos(qx/2)\sqrt{G(x,b)}\Big/\cos^2(qb/2)\end{array}\right]+\cos(qb)\psi_1(x)$$

(4.6)

$$\psi_3(x)=\left[\frac{\cos(qx/2)\sqrt{G(x,b)}-\sqrt{2}\cos^2(qb/2)X_1(x)}{q}\right] \qquad (4.7)$$

$$\psi_4(x)=\frac{G^2(0,b)}{4\sqrt{2}q}\left[\cos^{-1}\left\langle\frac{\sin(qx/2)}{\sin(qb/2)}\right\rangle+\frac{\sin qx/2}{2\sin^4(qb/2)}\left\langle\frac{G(0,b)-2G(0,x)}{\sqrt{2}}\right\rangle\sqrt{G(x,b)}\right]+\cos(qb)\psi_3(x)$$

(4.8)

FLUX

Now we calculate flux across y = 0 ,

$$\frac{\partial}{\partial y}T(x,0^+)\ ,\quad \text{for}\quad b\le|x|\le a \qquad \text{(From 2.1)}$$

We get

$$\frac{\partial T}{\partial y}(x,0^+)=\sum_{n=1}^{\infty}\alpha_n\varphi_n\cos\alpha_n x+\sum_{n=1}^{\infty}\alpha_n\left[\frac{a_1a_3-a_1a_4-a_2a_3}{a_1a_4+a_2a_3}\right]\varphi_n\cos\alpha_n x \qquad (4.9)$$

Now we use (2.15) in above relation, we get flux as-

$$\frac{\partial T}{\partial y}(x,0^+)=\int_0^b\frac{g(t)\sin qt}{G(x,t)}dt+\int_0^b g(\alpha)M(\alpha,x)d\alpha \qquad (4.10)$$

With

$$M(\alpha,x)=\sum_{n=1}^{\infty}\left[\frac{a_1a_3-a_1a_4-a_2a_3}{a_1a_4+a_2a_3}\right]\sin(\alpha\alpha_n)\cos\alpha_n x \qquad (4.11)$$

The first term in right hand side of (4.10) gives singularity while second term does not. We evaluate the integrals, after using (3.15) in (4.10)

$$\frac{\partial T}{\partial y}(x,0^+)=\frac{\sqrt{2}}{a}\psi_0\sum_{r=1}^{5}e_r\theta_r(x)t\int_0^b g(\alpha)M(\alpha,x)d\alpha\ ,\quad b<x\le a \qquad (4.12)$$

$$\theta_1(x)=\frac{a}{\sqrt{2}}-\theta_{01}(x)\ ,\quad \theta_{01}(x)=\frac{\sin(qx/2)}{\sqrt{G(b,x)}} \qquad (4.13)$$

$$\theta_2(x)=-\frac{aG(0,b)}{2\sqrt{2}}+\theta_{11}(x)\,,\qquad \theta_{11}(x)=\frac{a}{\sqrt{2}}\cos(qx)-\theta_{12}(x) \qquad (4.14\ a)$$

$$\theta_{12}(x)=\cos(qx)\theta_{01}(x)\theta_3(x)=\theta_{31}(x)+\theta_{32}(x) \qquad (4.14\ b)$$

$$\theta_{31}(x)=\frac{-aG(0,b)}{8\sqrt{2}}\left[4G(x,b-G(0,b)\right]+\frac{a}{\sqrt{2}}\cos(qx) \quad (4.15\text{ a})$$

$$\theta_{32}(x)=\cos(qx).\theta_{12}(x) \quad (4.15\text{ b})$$

$$\theta_4(x)=\sin^2 q\theta_{41}(x)+\theta_{42}+\theta_{43}\cos(qx) \quad (4.16\text{ a})$$

$$\theta_{42}=\frac{\sqrt{2}}{q}\left[2\sin(qb/2)+\cos(qb)\cosh^{-1}(\sec(qb/2)\right]$$

$$\theta_{43}=\frac{\sqrt{2}}{q}\cosh^{-1}\langle\sec qb/2\rangle \quad (4.16)\text{ b}$$

$$\theta_{41}(x)=\frac{\sec^2(qx/2)\sec(qb/2)}{2q\sqrt{G(b,x)}}\log\left|\frac{\sin(qb/2)+\sqrt{\frac{G(b,x)}{2}}}{\sin(qb/2)-\sqrt{\frac{G(b,x)}{2}}}\right| \quad (4.16\text{ c})$$

$$\theta_5(x)=\theta_{51}+\cos(qx)\theta_4(x) \quad (4.17)$$

$$\theta_{51}=\frac{\cos^2(qb/2)}{q\sqrt{2}}\left[\sin(qb/2)\left\langle 5+2\tan^2 qb/2\right\rangle-4\tan(qb/2)\sec(qb/2)\ -(4-3\cos^2(qb/2)\cosh^{-1}(\sec qb/2)\right]$$

Thus we see that flux possess square root singularity at crack tips.We define flux intensity factor at crack tips.

FLUX INTENSITY FACTOR

It is defined as

$$F_b=\lim_{x\to b^+}\sqrt{x-b}\left\langle\frac{\partial T}{\partial y}(x,0^+)\right\rangle \quad (4.18)$$

Now we use (4.12) in (4.18) and evaluate limit, we get

$$F_b=\psi_0\sqrt{\frac{2\tan(qb/2)}{\pi a}}\left[-e_1-e_2\cos qb+e_3\cos^2 qb\right] \quad (4.19)$$

0ther terms from flux, given by (4.10)-(4.17), do not possess singular term at x=b.

DISCUSSION AND CONCLUSION

The problem of disturbance in heat distribution caused due to a Griffith crack in an isotropic rectangle with most general boundary condition had been discussed. The general problem is reduced to Fredholm integral equation of second kind. The solution of Fredholm integral equation is obtained by approximation the kernel for prescribed (known) temperature and flux at boundary.

Method can be used for most general conditions also.

We used Fourier series method with the principle of cross-linear superposition. It is observed that the temperature is smooth at crack tips while flux is singular. It has Cauchy type of singularity.

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